

# Satellite Attitude Control Based Adaptive sliding Mode Method

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## Abstract

*This paper proposes an adaptive sliding-mode controller design for a three-axis stabilized rigid satellite attitude system with uncertain disturbances. The rigid satellite attitude control systems can be described by the dynamic equations and kinematic equations. This method combined sliding mode control and an adaptive algorithm, which is used to estimate the disturbances uncertainties. Tracking performance was guaranteed, as well as the stability of closed-loop attitude control systems which was analysed by using Lyapunov approach stability. The simulations results demonstrated the effectiveness of the proposed control method.*

**Keywords:** Adaptive controller; Attitude control; Rigid spacecraft; Sliding mode.

## I. Introduction

The attitude control of satellites had been a raising issue for decades [1-3]. Regarding the demand of future space missions, more and more accurate attitude controllers are required in order to meet the high-precision and high-stability of various spacecrafts. Moreover, unknown external and internal disturbances seriously affect attitude control performance. These problems pose a real challenge for attitude control system engineers. In fact to improve robust performance and control accuracy suitable control schemes are required to ensure the success of satellite mission.

Over the last years, many researchers had conducted extensive studies on the spacecraft attitude control system hence various controllers had been proposed to overcome this problem. These controllers included PID controller [4] and an historical overview of optimal control theory in the design of aerospace systems had been presented in [5-7], while the closed loop stability analysis was performed for Lyapunov's method. Sliding mode control results could offer high levels of robustness in the presence of parameter uncertainty and dynamic model errors. The first controller design of satellite attitude manoeuvres using sliding mode theory using rigid body equipped with three axis external control torques had been proposed in [8]. This had motivated a general sliding-mode scheme in [9] based on nonlinear sliding manifolds and Gibbs vector parameterisation of spacecraft

attitude. However this design procedure resulted in discontinuous controls that might be placed with saturated controls in order to avoid chattering. An important practical aspect of sliding mode was accounting for the control torque saturation in [10] where an asymptotically stable control method for robust attitude stabilisation was taken into account control input saturation explicitly. Nevertheless, this work provided stabilisation of spacecraft angular rate only. However these limitations had been removed subsequently in [11] such that a continuous version of the sliding mode control design of [12] was applied on a full state Lyapunov analysis.

An interesting approach to deal with complex plant dynamics that are changing or uncertain is the adaptive control which has received an increased attention in aerospace engineering. In fact all spacecraft dynamics models are subject to uncertainty such as satellite inertia matrix uncertainty and environmental disturbances, an adaptive controller for the International Space Station (ISS) was performed in [13] where the spacecraft had been considered as a rigid body equipped with a momentum exchange devices and the controller was developed incorporating gain-scheduled adaptation of the attitude gains to ensure acceptable attitude tracking performance. The inverse optimal theory and derive the adaptive control law had been employed in [14] to solve the attitude tracking control problem of a rigid spacecraft with external disturbances and an

uncertain inertia matrix. Also to compensate the parameter uncertainties and robust tracking performance, a robust adaptive PID-type controller had been proposed in [15] by incorporating a fuzzy logic system and a sliding-mode control. A separation-type principle for observer-based adaptive attitude tracking control of a fully actuated rigid spacecraft had been considered in [16].

In the present paper, an adaptive sliding mode control method was proposed for rigid spacecraft attitude control with uncertain disturbances. The method combined adaptive with sliding mode control to eliminate uncertain disturbances, and the stability of the system under disturbance was proved by Lyapunov method. Finally the simulations results showed that the proposed method could achieve an attitude stabilization control of satellite with uncertain disturbances.

## II. Mathematical model of satellite attitude

Mainly there are two methods for describing satellite attitude motion: Euler angle and quaternion, which can be inter converted. The dynamic and kinematic equation of satellite attitude can be expressed as follow: [17, 18]

$$J\dot{\omega} = -\omega^\times J\omega + u + d \quad (1)$$

$$\dot{q}_v = \frac{1}{2}(q_v^\times + q_0 I_3)\omega \quad (2)$$

$$\dot{q}_0 = -\frac{1}{2}q_v^T \omega \quad (3)$$

With  $J \in R^3$  denoting the constant and symmetrical moment of inertia matrix,  $\omega = [\omega_1, \omega_2, \omega_3]^T$  is the angular velocity vector of a body coordinate system relative to inertial coordinate system,  $u$  and  $d$  are satellite's control force and disturbances, respectively,  $I_3$  is three unit matrix,  $q_v = [q_1, q_2, q_3]^T$  is the vector part of  $q$ , the unit quaternion of satellite body coordinate system, and  $q = [q_0, q_v]^T$ , They are related as:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (4)$$

$[a^\times]$  is an operator on any vector  $a = [a_1, a_2, a_3]^T$  such that [19]:

$$[a^\times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \quad (5)$$

Let  $q_e = [q_{ve}, q_{0e}]^T$  denote relative attitude error from a desired reference frame to the body-fixed reference frame of the satellite, then:

$$q_e = q \otimes q_d^{-1} \quad (6)$$

with  $q_d^{-1}$  the inverse of the desired quaternion and  $\otimes$  is the quaternion multiplication operator. Therefore, the relative attitude error is obtained by:

$$\begin{bmatrix} \dot{q}_{ve} \\ \dot{q}_{0e} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_{0v} I_{3 \times 3} + (q_{ve}^\times) \\ -q_{ve}^T \end{bmatrix} \omega_e(t) \quad (7)$$

and

$$\omega_e = \omega - \omega_d \quad (8)$$

Where  $\omega_d$  represents the desired angular velocity of the body, assumed to be equal to zero, therefore

$$\omega_d = 0 \rightarrow \omega_e = \omega.$$

Hence, the rate of angular velocity can be obtained as follow:

$$\dot{\omega}_e = \dot{\omega} = -J^{-1}(\omega^\times)J\omega + J^{-1}u + J^{-1}d \quad (9)$$

The orbit of a small satellite, encompassing eccentricity, inclination, and altitude and the satellite in complex environments may be affected by aerodynamic moments  $d_p$ , gravity gradient moments  $d_g$ , geomagnetic moments  $d_m$  and solar pressure  $d_s$  [20]. The total environmental disturbance torque can be expressed as:

$$d = d_p + d_g + d_m + d_s \quad (10)$$

These moments of external disturbance do not always exist and remain unchanged, but are related to the altitude of the satellite orbit, the distribution of the structure and the conditions of the space environment. Assuming that the total disturbance torque  $d$  is bounded and is slow-varying, leads reasonably to  $\dot{d} \approx 0$ .

## III. Sliding-mode adaptive attitude controller design

In this section an adaptive sliding mode attitude controller was designed to compensate the effect of the variation of the spacecraft's disturbances. For this purpose, the controller law is shown in Eq. (11) and it can stabilize the origin of the plant (Eq. (1), (2) and (3)):

$$u = \omega^{\times}(J\omega) - C_1 J \dot{q}_{ve} - \hat{d} - k_1 \text{sat}(s) - k_2 s \quad (11)$$

where  $\hat{d}$  is the estimate of the disturbance  $d$ ,  $k_1$  and  $k_2$  are positive constant,  $\text{sat}(s)$  is the saturation function defined by following equation:

$$\text{sat}(s) = \begin{cases} \frac{s}{\|s\|} & \text{for } \|s\| \geq \varepsilon \\ \frac{s}{\varepsilon} & \text{for } \|s\| < \varepsilon \end{cases} \quad (12)$$

and  $s = [s_1, s_2, s_3]^T$  is a linear sliding surface in vector form which can be defined as:

$$\dot{s} = \dot{\omega} + C_1 \dot{q}_{ve} \quad (13)$$

Where  $C_1 = \text{diag}[c_1, c_2, c_3]$  with  $c_i > 0, i = 1, 2, 3$  is a scalar.

The derivative of the sliding surface combined with Eq. (7) and Eq. (9) led to the following:

$$\begin{aligned} \dot{s} &= \dot{\omega} + C_1 \dot{q}_{ve} \\ &= -J^{-1}(\omega^{\times})J\omega + J^{-1}u + J^{-1}d \\ &\quad + \frac{1}{2}C_1(q_{0e}I_{3 \times 3} + (q_{ve}^{\times}))\omega \end{aligned} \quad (14)$$

Then

$$J\dot{s} = -(\omega^{\times})J\omega + u + d + JC_1 \dot{q}_{ve} \quad (15)$$

In order to prove the stability of the system, a Lyapunov candidate function could be considered as:

$$V = \frac{1}{2} s^T J s + \frac{1}{2} C_2 e^T e \quad (16)$$

Where  $e = d - \hat{d}$  is the disturbance estimation error, thus  $\dot{V}$  is given by:

$$\begin{aligned} \dot{V} &= s^T J \dot{s} + C_2^{-1} e^T \dot{e} \\ &= s^T [-\omega^{\times}(J\omega) + u + d + C_1 J \dot{q}_{ve}] - C_2^{-1} e^T \dot{\hat{d}} \\ &= s^T e - k_1 \|s\| - k_2 \|s\|^2 - C_2^{-1} e^T \dot{\hat{d}} \end{aligned} \quad (17)$$

The derivative of the disturbance observer can be designed as follow:

$$\dot{\hat{d}} = C_2 s \quad (18)$$

Hence (17) leads to

$$\begin{aligned} \dot{V} &= -k_1 \|s\| - k_2 \|s\|^2 \\ &\leq 0 \end{aligned} \quad (19)$$

Since  $\dot{V}$  is negative definite then the asymptotic stability has been proved.

#### IV. Numerical simulations

To demonstrate the effectiveness of the proposed control schemes, numerical simulations results have been carried out under the Matlab/Simulink platform. These results were obtained using the following parameters.

Table1. Satellite simulation parameters

Parameter	Value
Inertia [ kg.m <sup>2</sup> ]	$\begin{bmatrix} 152.9 & 0 & 0 \\ 0 & 152.5 & 0 \\ 0 & 0 & 4.91 \end{bmatrix}$
Orbit [ km ]	686
Inclination [ deg ]	98
Initial attitude [ deg ]	[-0.15 0.2 0.4]
Initial attitude rate [ deg/ s ]	[0 -0.06 0]
External Torques [ N.m ]	$(\ \omega\ ^2 + 0.005) \begin{bmatrix} 0.05 \sin(0.8t) \\ 0.05 \cos(0.5t) \\ 0.05 \cos(0.3t) \end{bmatrix}$

Parameters were selected as  $k_1 = 0.112, k_2 = 8.5, C_1 = \text{diag}(0.75, 0.85)$  and  $C_2 = \text{diag}(10e3 * (3.75, 1.96, 1.85))$

The control objective was to transfer the system from the initial attitude to the desired attitude, the simulation results are shown in Figures 1-7.

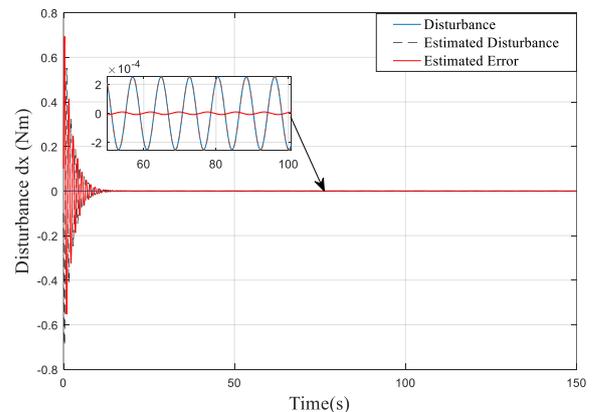


Figure. 1 Time responses of Disturbance dx

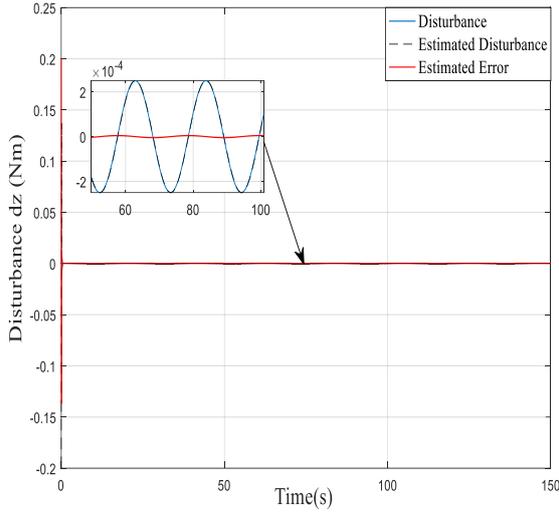


Figure. 2 Time responses of Disturbance dy

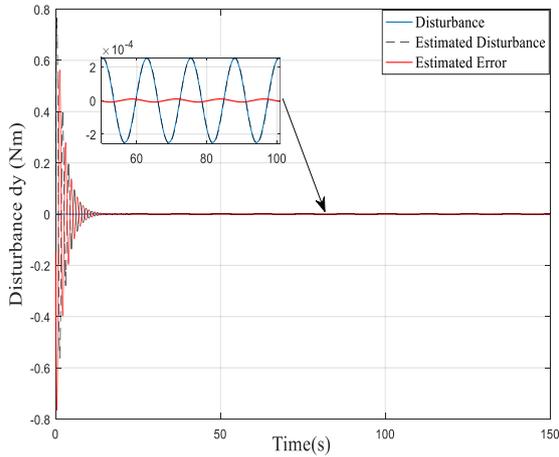


Figure. 3 Time responses of Disturbance dz

Figures 1-3 show the time evolutions of disturbances, disturbances estimations and estimation error according the three axes respectively. It is clear that the proposed controller could effectively estimate the uncertain disturbances. The control torques of three axes is given in Fig. 4. It is seen that at beginning of the simulation the control torque was larger due to the fact that the proposed controller had better dynamic response than a classical control and could estimate the compensate for the disturbances. Furthermore, after 7.2s, no excessive control energy was required to reject the disturbances.

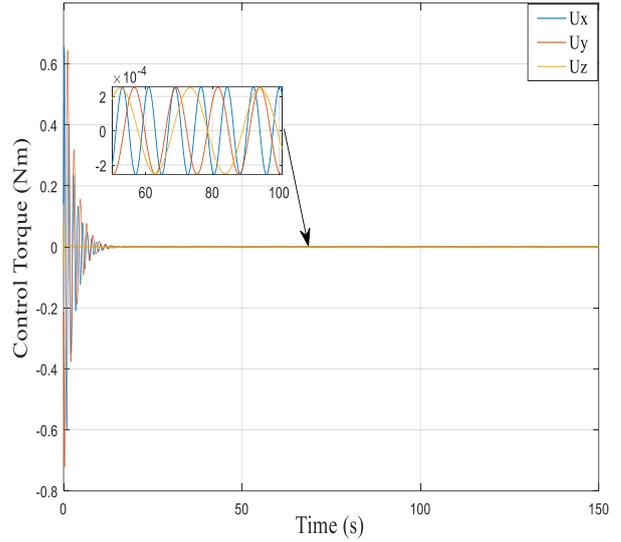


Figure. 4, Time response of control Torque

Fig. 5 and Fig. 6 illustrate the attitude control error and the error quaternions where the attitude control accuracy was improved and the partial amplifications demonstrated that the error has been reduced to the lowest.

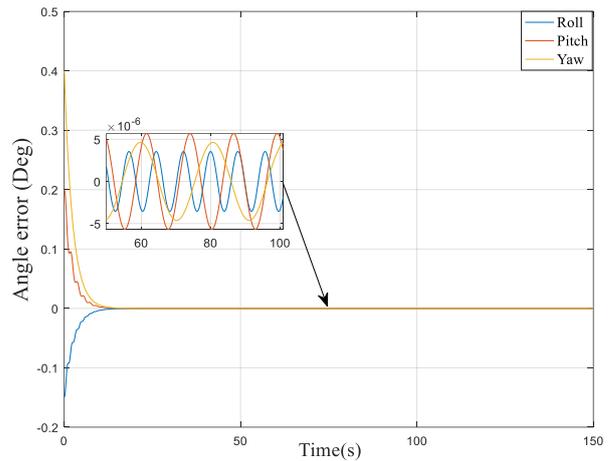


Figure. 5, Attitude angle error

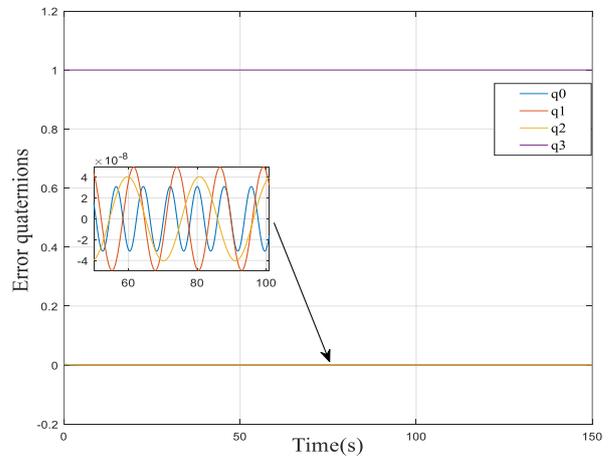


Figure. 6, Error quaternions

The responses of the spacecraft error angular velocity components are depicted in Fig 7. It can be seen that the stabilization was obviously improved using the proposed controller.

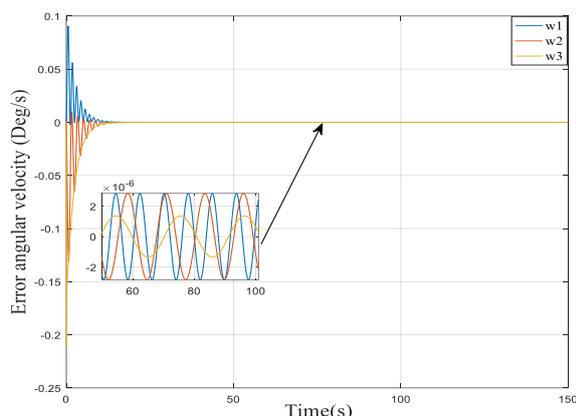


Fig. 7. Error angular velocity

## V. Conclusion

The main goal of this paper was to focus on the adaptive sliding mode controller design of microsatellite attitude control with uncertain disturbances; so that the controller could successfully improve the pointing accuracy and the stability of the system and also to prove theoretically that it was based on Lyapunov theory as confirmed through simulation.

Numerical simulations showed that the controller designed in this paper achieved better the pointing accuracy and the stabilization of the spacecraft. Moreover, the error could be reduced to the lowest level by the proposed controller. This method provided a useful and promising way for the attitude control of rigid spacecraft.

For the future research work, it is required to take into account the model with the flexible vibrations and to design a new controller to achieve better control effect.

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